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MATDIP301

Third Semester B.E. Degree Examination, Jan./Feb. 2023
Advanced Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting at least TWO full questions from each part.***

PART – A

1 a. Express $\sqrt{3}+i$ in the polar form and hence find its modulus and amplitude. **(06 Marks)**

b. Find all the values of $\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)^{3/4}$. **(07 Marks)**

c. Expand $\cos^6\theta$ in a series of cosines of multiples of θ. **(07 Marks)**

2 a. Find the n^{th} derivative of $e^{ax}\cos(bx+c)$. **(06 Marks)**

b. If $y = a\cos(\log x) + b\sin(\log x)$, show that $x^2y_{h+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$. **(07 Marks)**

c. Find the n^{th} derivative of $\cos x \cos 3x$. **(07 Marks)**

3 a. With the usual notation prove that $\tan\varphi = r\frac{d\theta}{dr}$. **(06 Marks)**

b. Find the angle between the curves $r^w = a^n \cos n\theta$ and $r^n = b^n \sin n\theta$. **(07 Marks)**

c. Obtain the Maclaurin's expansion of $f(x) = \tan x$ upto third degree terms. **(07 Marks)**

4 a. State and prove Euler's theorem. **(06 Marks)**

b. If $z = f(x, y)$ with $x = e^u \sin v$, $y = e^u \cos v$ prove that $\left(\frac{\partial z}{\partial u}\right)^2 + \left(\frac{\partial z}{\partial v}\right)^2 = e^{2u}\left\{\left(\frac{\partial z}{\partial v}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2\right\}$. **(07 Marks)**

c. If $x = r\cos\theta$, $y = r\sin\theta$, then prove that $JJ^1 = 1$. **(07 Marks)**

PART – B

5 a. Obtain a reduction formula for $\int_0^{\pi/2} \cos^n x\, dx$ where n is the integer. **(06 Marks)**

b. Evaluate by changing the order of integration $\int_0^1\int_0^x (xy + y^2)\, dx dy$. **(07 Marks)**

c. Evaluate $\int_0^1\int_0^1\int_0^2 xyz\, dz\, dy\, dx$. **(07 Marks)**

6 a. With the usual notation prove that
$$\beta(m,n) = \frac{\Gamma(m).\Gamma(n)}{\Gamma(m+n)}.$$ (06 Marks)

b. Prove that $\int_{-1}^{1}(1+x)^{m-1}(1-x)^{n-1}dx = 2^{m+n-1}\beta(m,n)$. (07 Marks)

c. Show that $\int_{0}^{\infty} e^{-x^2}dx = \frac{1}{2}\sqrt{\pi}$. (07 Marks)

7 Solve:

a. $(xy - x)\,dx + (xy + y)\,dy = 0$. (06 Marks)

b. $\frac{dy}{dx} = \frac{xy - y^2}{x^2}$. (07 Marks)

c. $(5x^4 + 3x^2y^2 - 2xy^3)dx + (2x^3y - 3x^2y^2 - 5y^4)dy = 0$. (07 Marks)

8 a. Solve $(D^2 - 6D + 9)\,y = e^x$. (06 Marks)

b. Solve $(D^2 - 6D + 10)\,y = \cos 2x$. (07 Marks)

c. $(D^2 - 2D + 1)^2 y = x^2 + x + 1$. (07 Marks)

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